INTRODUCTION

It is known that open damping sites for household garbage have been used since time immemorial in many countries. [1]. This issue is devoted to an extensive scientific literature analyzing the shortcomings inherent in the open dumping sites [2]. It is also known that in the early thirties of the last century in USA it was invented and spread a way of storage and processing by means of landfills [3],[4],[5]. In the Fig.1 is depicted a schematic of a state-of-the-art modern landfill. Today a lot of experts agree that sanitary landfilling may solve the problem of the urban trash [1],[2].

The purpose of landfill is to bury the trash in such a way that it will be isolated from groundwater, will be kept dry and will not be in contact with air. Landfilling has the disadvantage needing extremely long time to fill its volume and attain an inert mode. Novelty of the present paper consists in taking into consideration the struggle for survival between the microbial populations of the acidogens and methanogens converting garbage in the carbon dioxide and methane. Present mathematical model constitutes a set of two first order nonlinear ordinary differential equations, six empirical parameters and two initial conditions. Equations were solved by means of the Runge-Kutta finite-difference numerical method implemented in the Matlab. The results are illustrated by numerical examples. The calculations show that there is a possibility for reduction runtime for the landfill from start to exit to inert mode.

ABSTRACT

A widely accepted method consists in use of the sanitary landfill which was developed in USA in the early thirties. The purpose of landfill is to bury the trash in such a way that it will be isolated from groundwater, will be kept dry and will not be in contact with air. Landfilling has the disadvantage needing extremely long time to fill its volume and attain an inert mode. Novelty of the present paper consists in taking into consideration the struggle for survival between the microbial populations of the acidogens and methanogens converting garbage in the carbon dioxide and methane. Present mathematical model constitutes a set of two first order nonlinear ordinary differential equations, six empirical parameters and two initial conditions. Equations were solved by means of the Runge-Kutta finite-difference numerical method implemented in the Matlab. The results are illustrated by numerical examples. The calculations show that there is a possibility for reduction runtime for the landfill from start to exit to inert mode.

Fig. 2 Carbon dioxide, methane and trace gases (% by volume) vs time.

Fig. 2 shows the idealized time-dependence curves for carbon dioxide, methane and trace gases as described in scientific literature. The following three stages could be recognized in the Fig.2 for methane curve: exponential growth, inert (with plateau) and decay. Curve of the carbon dioxide is more complicated combination of the Bell-shaped form with plateau and decay curves. Attention should be drawn to the intersection point for the above curves which is of the important physical meanings. To the left of the intersection point between carbon dioxide and methane graphs, the curve for carbon dioxide is higher than the curve for methane. To the right of the intersection point, the curve for carbon dioxide is lower than the curve for methane. In our opinion, intersection point is a critical point of the process. It separates the regions of prevalence for gases: carbon dioxide and methane, respectively. More to the right of this point on the graphs, two curves become parallel. From the physical point of view it means that process has entered the inert stage which later will be replaced by a decay stage.

ESTABLISHMENT OF OBJECTIVES

It can take an extremely long time for landfill to attain the inert stage, for example 50-70 years. This is a drawback of the landfilling process. To reduce this time indeed is a key problem for many industrial, environmental and medical issues. Thus the main objectives of the present paper is obtaining insight on the way to diminish extremely long landfill time to become inert. We suggest that this objective could be obtained by taking into consideration the behaviour of the acidogens and methanogens bacterial populations curves right of the above crossing point.

Fig. 1. Schematic cross-section of a landfill

The landfill construction starts from a long term using receive pit. The pit bottom should be covered by thick clay layer with the aim of preventing the penetration of contaminated water into the surrounding soil. During the construction the trash is poured into the pit according with daily limitations. Cells are carefully insulated by means of clay. Totally filled pit also insulated by clay. The method implies landfill gas creation from the urban waste by means of the decomposition process [6],[7],[8],[9]. According to the obtained data landfill gas is composed of a mixture of different gases. By volume it typically consists mainly from methane and carbon dioxide. Also it includes small amounts of nitrogen, oxygen, hydrogen and other gases. Landfill gas is the valuable source of energy for the power plants, thus landfills convert “waste to watts”.

KEYWORDS: landfill, waste processing, mathematical model, runtime reduction.
MATERIALS AND METHODS

In current paper mathematical modeling combined with the Lotka-Volterra theory of the biological competition are used to obtain the insight on landfill internal processes resulting in the generation of the methane and carbon dioxide. Equations are written as follows [10]:

\[
\frac{dN_1}{dt} = r_1 N_1 \left( 1 - \frac{N_1}{K_1} - b_{12} \frac{N_2}{K_1} \right) \\
\frac{dN_2}{dt} = r_2 N_2 \left( 1 - \frac{N_2}{K_2} - b_{21} \frac{N_1}{K_2} \right)
\]

If \( t = 0 \) initial conditions are:

\[ N_1(0) = N_{1,0}, \quad N_2(0) = N_{2,0} \]

Here \( N_1 \) and \( N_2 \) are the amounts of the first and the second competitive populations, \( r_1, r_2 \) are the linear birth rates, \( K_1, K_2 \) are the carrying capacities, \( b_{ij} \) are measure of the possible effect of \( i \) on \( j \) and on \( j \). Solution was obtained by means of the Runge-Kutta solver ode45 which was implemented in Matlab computational software.

TEST SOLUTION

As a test for the numerical method problem from the mathematical biology about the struggle for survival between two populations of yeasts is under consideration. The test problem is described as a set of the coupled ordinary differential equations with six parameters and two initial conditions.

Lotka-Volterra equations for the test problem are given as:

\[
\frac{dX}{dt} = AX - BX^2 - CXY \\
\frac{dY}{dt} = DY - EY^2 - FXY
\]

where,

\[ A = 0.25860, \quad B = -0.02030, \quad C = -0.05711, \quad D = 0.0574, \quad E = -0.00977, \quad F = -0.00480, \]

\[ X(0) = 0.418, \quad Y(0) = 0.6315 \]

Thus applied Runge-Kutta method indeed is viable here and two cases were treated by this method.

CASE 1

The set of parameters used on the model is given as:

\[ K_1 = 100, \quad K_2 = 100, \quad r_1 = 0.5, \quad r_2 = 0.5, \quad b_{12} = 0.6, \quad b_{21} = 0.67 \]

The solution is given by two curves depicted in the Fig.6 that have a point of the intersection. Left of this crossing both curves converge. Right of this crossing both curves diverge. After some time, the approach closes and the curves turn into two straight lines parallel to the horizontal axis of coordinates, which maintain a constant distance. This means that the landfill goes to the inert mode. It is possible to determine the beginning of the inert mode by constructing a tangents to the both graphs. In this case, it can be concluded from Fig. 5 that the landfill reaches the fully developed inert stage for \( t = 40 \) years.

CASE 2

Present solution for two kinds of yeasts is given in the Fig.3. It is exactly the same as given below in the Fig.4 following the work [11].
The coefficients used to describe the case 2 in the Fig.6 are

\[ K_1 = 100, K_2 = 100, r_1 = 0.5, r_2 = 0.5, \text{ and additionally} \]

\[ a_{12} = 0.72, a_{21} = 0.80. \]

The last two coefficients are greater than the values of similar coefficients accepted for case case 1. In present case, both intraspecic and interspecic populations struggle occur more intensively. Accordingly, the release of the waste into an inert mode occurs for a longer time with comparison to the case 1. Thus territory occupied by landfill can be used for the different economic needs at \( t=70 \) year after start of the landfill work.

CONCLUSIONS

Sanitary Landfill is a term used for the name of an engineering facility for the storage and processing of waste. In this structure, as a result of the process of decomposition of waste under the influence of bacteria asidogens and methanogens and others, generation of methane, carbon dioxide and associated trace components occurs. Thus, a landfill gas is formed which is a valuable source of energy. At the same time, as practice shows, landfilling also has the disadvantage of needing extremely slow (on the order of several decades) time to fill its volume and attain an inert mode. In this paper, a new approach is developed consisting of using the biological theory of populations competition to obtain insight on the complex processes occurring in the volume or the landfill. The calculations based on the bio-mathematical modelling show the possibility for reduction the runtime of the landfill from start to exit to inert mode by means of the changing parameters \( \alpha_{12}, \alpha_{21} \) which are important elements of the Lotka-Volterra model for two species competition.

REFERENCES


